



Problem of the Week

Grade 7 and 8

Prime Picking Solution

Problem

A natural number greater than 1 is said to be *prime* if its only factors are 1 and itself. For example, the number 7 is prime since its only factors are 1 and 7. A *perfect square* is an integer created by multiplying an integer by itself. The number 25 is a perfect square since it is 5×5 or 5^2 . Determine the smallest perfect square that has three different prime numbers as factors.

Solution

The problem itself is not very difficult once you determine what it is asking. So before looking at the solution, let us examine some perfect squares.

The numbers 4 and 9 are both perfect squares that have only one prime number as a factor, $4 = 2^2$ and $9 = 3^2$.

The number 36 is a perfect square since $36 = 6^2$. However, the number $6 = 2 \times 3$ so $36 = (2 \times 3)^2 = 2 \times 3 \times 2 \times 3 = 2^2 \times 3^2$. So 36 is the product of the square of each of two different prime numbers. In fact, since 2 and 3 are the smallest prime numbers, 36 is the smallest perfect square that has two different prime factors. In order to create a perfect square we must find the product of the squares of prime numbers.

Using this idea, we can create the smallest perfect square with three different prime factors by squaring each of the three smallest primes, 2, 3, and 5, and then multiplying these squares together. So the smallest perfect square that uses three different prime factors would be $2^2 \times 3^2 \times 5^2 = 4 \times 9 \times 25 = 900$. It should be noted that $900 = 30^2 = (2 \times 3 \times 5)^2$.

\therefore the smallest perfect square with three different prime factors is 900.

