



Problem of the Week Grade 9 and 10

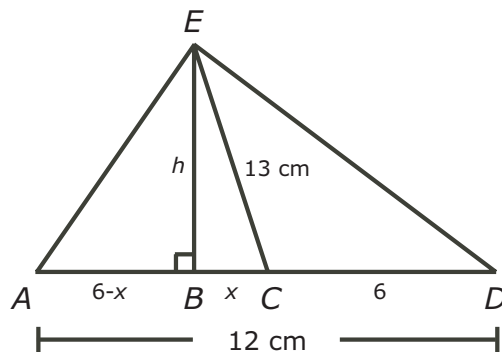
Traversing the Triangle Solution

Problem

A *median* is a line segment drawn from the vertex of a triangle to the midpoint of the opposite side. A triangle has an area of 72 cm^2 . The length of one side is 12 cm and the length of the median to this side is 13 cm . Determine the length of the other two sides of the triangle.

Solution

Start with a diagram to represent the problem.



Let AD be the 12 cm side and EC be the 13 cm median drawn to that side. Draw the altitude from E to side AD meeting it at B . Let EB be h .

Since EC is a median, $AC = CD = \frac{1}{2}(AD) = \frac{1}{2}(12) = 6 \text{ cm}$.

Let BC , the distance along AD from the altitude to the median, be x .

Then $AB = 6 - x$.

The area of the triangle is 72 cm^2 so $\frac{AD \times EB}{2} = 72$. Then $\frac{12h}{2} = 72$ and $h = 12 \text{ cm}$ follows.

$\triangle EBC$ is right angled so, using Pythagoras' Theorem,

$$x^2 = 13^2 - h^2 = 13^2 - 12^2 = 169 - 144 = 25 \text{ and } x = 5 \text{ cm } (x > 0).$$

Then $BD = BC + CD = x + 6 = 11 \text{ cm}$ and $AB = 6 - x = 1 \text{ cm}$.

$\triangle EAB$ is right angled so, using Pythagoras' Theorem, $EA^2 = EB^2 + AB^2 = 12^2 + 1^2 = 145$ and $EA = \sqrt{145} \text{ cm}$. ($EA > 0$)

$\triangle EBD$ is right angled so, using Pythagoras' Theorem,

$$ED^2 = EB^2 + BD^2 = 12^2 + 11^2 = 144 + 121 = 265 \text{ and } ED = \sqrt{265} \text{ cm } (ED > 0).$$

Therefore the lengths of the other two sides are $\sqrt{145} \text{ cm}$ and $\sqrt{265} \text{ cm}$. These lengths are approximately 12.0 cm and 16.3 cm .

