

Problem of the Week

Grade 9 and 10

The Key Factor

Solution

Problem

A specific type of six-digit number is formed by repeating a three-digit number, for example, 265 265 or 325 325 or 143 143. Determine the largest integer which will divide all such numbers.

Solution

To get started look at the prime factorization of each of the given numbers.

$$265\ 265 = 5 \times 53\ 053 = 5 \times 7 \times 7\ 579 = 5 \times 7 \times 11 \times 689 = 5 \times 7 \times 11 \times 13 \times 53$$

$$325\ 325 = 5 \times 65\ 065 = 5 \times 5 \times 13\ 013 = 5 \times 5 \times 7 \times 1\ 859 = 5 \times 5 \times 7 \times 11 \times 169 = 5 \times 5 \times 7 \times 11 \times 13 \times 13$$

$$143\ 143 = 7 \times 20\ 449 = 7 \times 11 \times 1\ 859 = 7 \times 11 \times 11 \times 169 = 7 \times 11 \times 11 \times 13 \times 13$$

All of the numbers are divisible by $7 \times 11 \times 13 = 1\ 001$. These are the only three factors common to all three numbers.

Pick another six-digit number formed by repeating a three-digit number and test to see if it is also divisible by 1 001. The number 246 246, for example, is $1\ 001 \times 246$. It would appear that our conjecture (guess) is correct but it has not been proven.

Let $abc\ abc$ be any six digit number formed by repeating the three-digit number abc .

$$\begin{aligned} \text{Then } abc\ abc &= abc000 + abc \\ &= 1000 \times abc + abc \\ &= 1000 \times abc + 1 \times abc \\ &= 1\ 001 \times abc \end{aligned}$$

Since $abc\ abc = 1\ 001 \times abc$, it is divisible by 1 001. A specific number $abc\ abc$ may also have other divisors but 1 001 is the largest divisor common to all such numbers. In the first example $265\ 265 = 1\ 001 \times 5 \times 53$ and in the third example $143\ 143 = 1\ 001 \times 11 \times 13$. Both numbers have other factors but no other common factors. In some cases there will be other common factors but not in general.

This problem is not hard if you initially “get it”. The solution presented shows an approach that can be taken when you may not be certain where to begin. Try some specific examples and then attempt to generalize based on what you observe from the specific examples. Also note that discovering that 1 001 worked for the three given examples and the test example is not sufficient to make a general conclusion that 1 001 divides all such numbers.

