Problem of the Week
Grade 7 and 8

Can You DIGIT?
Solution

Problem

The three digit number $5A4$ is divisible by 4 and the three digit number $37B$ is divisible by 3. Determine the largest positive difference between $5A4$ and $37B$.

Solution

The largest positive difference will occur when $5A4$ is a large as possible and $37B$ is as small as possible. We are therefore looking for the largest possible value of A and the smallest possible value of B.

A number is divisible by 4 if the last two digits of the number are divisible by 4. For $5A4$ to be divisible by 4, the only possible values of A are 0, 2, 4, 6, and 8 since 04, 24, 44, 64, and 84, respectively, are the only two digit numbers that end in 4 and are divisible by 4. (04 is technically not a two-digit number but a larger number could end in these two digits.) Since we want $5A4$ to be as large as possible, $A = 8$ and $5A4 = 584$.

A number is divisible by 3 if the sum of its digits is divisible by 3. For $37B$ to be divisible by 3, the only possible values of B are 2, 5, and 8.

- When $B = 2$ the number is 372. It is divisible by 3 since $3 + 7 + 2 = 12$ is divisible by 3.
- When $B = 5$ the number is 375. It is divisible by 3 since $3 + 7 + 5 = 15$ is divisible by 3.
- When $B = 8$ the number is 378. It is divisible by 3 since $3 + 7 + 8 = 18$ is divisible by 3.

No other three digit numbers of the form $37B$ will be divisible by 3 since no other values of B, other than 2, 5, and 8, give a digit sum that is divisible by 3. Since we want $37B$ to be as small as possible, $B = 2$ and $37B = 372$.

The largest positive difference occurs when $A = 8$ and $B = 2$ so that $5A4 - 37B = 584 - 372 = 212$.

$\therefore$ **the largest positive difference is 212.**