

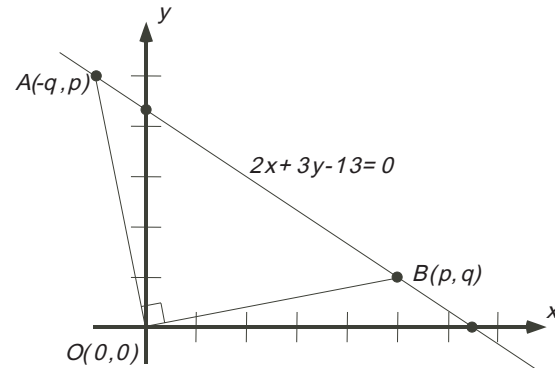


Problem of the Week Grade 11 and 12

Get Your Analytic Skills On Solution

Problem

OAB is an isosceles right triangle with vertex O at the origin $(0,0)$, vertices A and B on the line $2x + 3y - 13 = 0$ and $\angle AOB = 90^\circ$. Determine the area of $\triangle OAB$.



Solution

In analytic geometry problems, a representative diagram is important and often provides clues for the solution of the problem. The diagram above has the given information plus a couple of pieces of information that will be justified now.

Let B have coordinates (p, q) . The slope of $OB = \frac{q}{p}$. Since $\angle AOB = 90^\circ$, $OB \perp OA$ and the slope of OA is the negative reciprocal of OB . Therefore the slope of $OA = \frac{p}{-q}$. Since the triangle is isosceles, $OA = OB$ and it follows that the coordinates of A are $(-q, p)$. (We can verify this by finding the length of OA and the length of OB and showing that both are equal to $\sqrt{p^2 + q^2}$.)

Since $B(p, q)$ is on the line $2x + 3y - 13 = 0$, it satisfies the equation of the line.

$$\therefore 2p + 3q - 13 = 0 \quad (1)$$

Since $A(-q, p)$ is on the line $2x + 3y - 13 = 0$, it satisfies the equation of the line.

$$\therefore -2q + 3p - 13 = 0 \text{ or } 3p - 2q - 13 = 0 \quad (2)$$

Since we have two equations and two unknowns, we can use elimination to solve for p and q .

$$(1) \times 2 \quad 4p + 6q - 26 = 0$$

$$(2) \times 3 \quad 9p - 6q - 39 = 0$$

$$\text{Adding, we obtain} \quad 13p - 65 = 0$$

$$\therefore p = 5$$

$$\text{Substituting in (1)} \quad 10 + 3q - 13 = 0$$

$$3q = 3$$

$$\therefore q = 1$$

The point B is $(5, 1)$ and the length of $OB = \sqrt{5^2 + 1^2} = \sqrt{26}$. Since $OA = OB$, $OA = \sqrt{26}$. $\triangle AOB$ is a right triangle so we can use OB as the base and OA as the height in the formula for the area of a triangle. Then the area of $\triangle AOB = \frac{OA \times OB}{2} = \frac{\sqrt{26} \sqrt{26}}{2} = 13$.

$\therefore$ the area of $\triangle AOB$ is 13 units².

