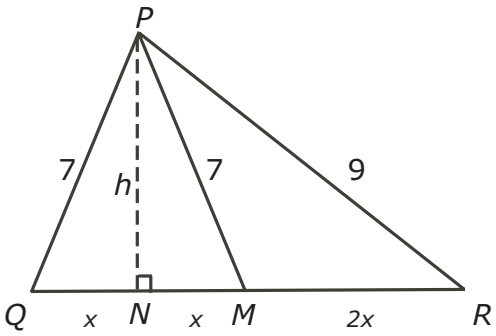
# Problem of the Week
## Grade 11 and 12

## Enough Information?
## Solution




### Problem

In $\triangle PQR$, $PQ = 7$, $PR = 9$ and median $PM = 7$. Determine the length of $QR$.

### Solution 1

Since $PQ = PM = 7$, $\triangle PQM$ is isosceles. In $\triangle PQM$, draw an altitude from $P$ to $QM$, intersecting at $N$. In an isosceles triangle, the altitude drawn to the base bisects the base. Therefore $QN = NM = x$. Since $PM$ is a median in $\triangle PQR$, $MR = QM = 2x$. Let $PN = h$.

$\triangle PNM$ is a right triangle. Using Pythagoras' Theorem,

$$\begin{aligned} PN^2 &= PM^2 - NM^2 \\ h^2 &= 7^2 - x^2 \\ h^2 &= 49 - x^2 \qquad (1) \end{aligned}$$

$\triangle PNR$ is a right triangle. Using Pythagoras' Theorem,

$$\begin{aligned} PN^2 &= PR^2 - NR^2 \\ h^2 &= 9^2 - (x + 2x)^2 \\ h^2 &= 81 - (3x)^2 \\ h^2 &= 81 - 9x^2 \qquad (2) \end{aligned}$$

In equations (1) and (2), the left side of each equation is $h^2$. Therefore, the right side of equation (1) must equal the right side of equation (2). So

$$\begin{aligned} 49 - x^2 &= 81 - 9x^2 \\ -x^2 + 9x^2 &= 81 - 49 \\ 8x^2 &= 32 \\ x^2 &= 4 \\ x &= 2, \quad x > 0 \end{aligned}$$

$\therefore QR = QN + NM + MR = x + x + 2x = 4x = 8$ units.

**A solution involving trigonometry and a system of equations is presented on the next page.**

**Problem**

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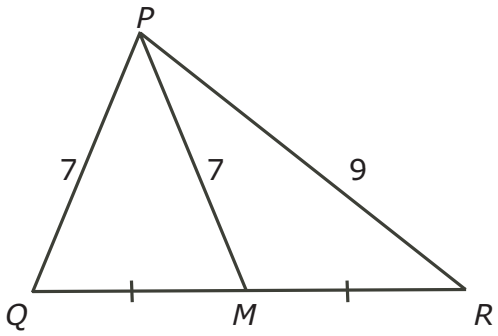


**Solution 2**

This solution is presented for students who have done some trigonometry and know the law of cosines.

Since $PM$ is a median, $QM = MR = x$. Then $QR = 2x$.

Using the law of cosines in $\triangle PQM$,

$$\begin{aligned} PM^2 &= PQ^2 + QM^2 - 2(PQ)(QM)\cos(Q) \\ 7^2 &= 7^2 + x^2 - 2(7)(x)\cos(Q) \\ 49 &= 49 + x^2 - 14x\cos(Q) \\ 14x\cos(Q) &= x^2 \qquad (1) \end{aligned}$$

Using the law of cosines in $\triangle PQR$,

$$\begin{aligned} PR^2 &= PQ^2 + QR^2 - 2(PQ)(QR)\cos(Q) \\ 9^2 &= 7^2 + (2x)^2 - 2(7)(2x)\cos(Q) \\ 81 &= 49 + 4x^2 - 28x\cos(Q) \\ 28x\cos(Q) &= 4x^2 - 32 \qquad (2) \end{aligned}$$

Using elimination to solve for $x^2$,

$$\begin{aligned} (1) \times 2 \qquad 28x\cos(Q) &= 2x^2 \\ (2) \qquad 28x\cos(Q) &= 4x^2 - 32 \\ \text{Subtracting} \qquad 0 &= -2x^2 + 32 \\ 2x^2 &= 32 \\ x^2 &= 16 \\ x &= 4, \quad x > 0 \\ QR = 2x &= 8 \end{aligned}$$

$\therefore$ the length of $QR$ is 8 units.