



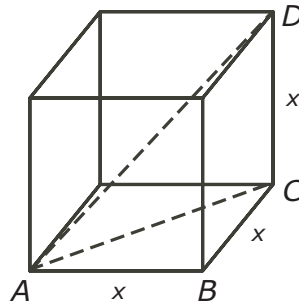
Problem of the Week

Grade 11 and 12

Sphere Pressure Solution

Problem

A cube rests inside a sphere so that each vertex touches the sphere. The radius of the sphere is 6 cm. Determine the volume of the cube.



Solution

Label four vertices of the cube A, B, C, D as shown in the diagram. Let x represent the side length of the cube. Then $AB = BC = CD = x$.

The diagonals of a cube intersect in a point such that the distance from the intersection point to each vertex is equal. Since each vertex of the cube touches the sphere, the diagonal of the cube, AD , is equal in length to the diameter of the sphere. Therefore $AD = 2(6) = 12$ cm.

Each face of a cube is a square so $\angle ABC = 90^\circ$. Using Pythagoras' Theorem,

$$AC^2 = AB^2 + BC^2 = x^2 + x^2 = 2x^2.$$

In a cube the sides are perpendicular to the base. In particular, DC is perpendicular to the base and it follows that $DC \perp AC$. Therefore $\triangle DCA$ is a right angled triangle. Using Pythagoras' Theorem,

$$AD^2 = AC^2 + CD^2 = 2x^2 + x^2 = 3x^2.$$

But $AD = 12$ so $AD^2 = 144$. Then $3x^2 = 144$, $x^2 = 48$ and $x = 4\sqrt{3}$ since $x > 0$. The volume of the cube is $x^3 = (4\sqrt{3})^3 = 192\sqrt{3}$ cm³.

$\therefore$ the volume of the cube is $192\sqrt{3}$ cm³.

