



Problem of the Week

Grade 9 and 10

Card Logic Solution

Problem

Four playing cards are placed in a row, from left to right. One card is a club (\clubsuit), one card is a diamond (\diamondsuit), one card is a heart (\heartsuit), and one card is a spade (\spadesuit), not necessarily in that order. The number on each card is different. Using the following clues, determine the exact order of the cards, from left to right, including the suit and number.

1. The club and the spade are immediately beside each other, in that order, left to right.
2. The diamond is not the first (leftmost) card, and the heart is not the fourth (rightmost) card.
3. The 7 is somewhere to the left of the heart and the 3 is somewhere to the right of the heart.
4. The two cards whose values are still unknown add to nine.
5. From left to right, the cards are arranged from largest to smallest.

Solution

Let C be the club, D be the diamond, H be the heart and S be the spade.

Using the first clue, we can determine that there are only three possible placements for C and S , namely $\{C, S, \square, \square\}$, $\{\square, C, S, \square\}$ or $\{\square, \square, C, S\}$.

We will use the second clue with each of the three possible placements. From $\{C, S, \square, \square\}$ we obtain: $\{C, S, H, D\}$ since H cannot go last. From $\{\square, C, S, \square\}$ we obtain $\{H, C, S, D\}$ since D cannot be first and H cannot be last. From $\{\square, \square, C, S\}$ we obtain $\{H, D, C, S\}$ since D cannot go first.

The third clue says that 7 is somewhere to the left of the heart. This means that the heart cannot be furthest to the left. Therefore, we can rule out two of the possibilities that we just found leaving $\{C, S, H, D\}$ as the only possible ordering of the suits. Using the third clue further, we know that a 7 is somewhere to the left of the heart and a 3 is somewhere to the right of the heart. We can conclude from this that the rightmost card is the 3 of diamonds. At this point we know the following: it is either $\{7C, S, H, 3D\}$ or $\{C, 7S, H, 3D\}$. We know the 7 is in either the first or second spot, the 3 of diamonds is in the fourth spot. We still do not know the other two numbers.

Using the fourth clue we know that the remaining pair of numbers add to 9. The numbers 7 and 3 are already used. Since numbers are arranged from highest to lowest and numbers are used only once, we can rule out the following pairs: 7 and 2, 6 and 3. That leaves only two possible pairs: 8 and 1, and 5 and 4. Using the fifth clue we can place the numbers to see if the solution is valid. Trying 8 and 1 in our possible solutions we obtain $\{7C, 8S, 1H, 3D\}$ or $\{8C, 7S, 1H, 3D\}$. Neither of these are valid since the card values would not be arranged largest to smallest. Trying 5 and 4 in our possible solutions we obtain $\{7C, 5S, 4H, 3D\}$ or $\{5C, 7S, 4H, 3D\}$. Only the first solution satisfies the condition that the numbers are arranged largest to smallest.

\therefore the cards are the 7 of clubs ($7\clubsuit$), the five of spades ($5\spadesuit$), the four of hearts ($4\heartsuit$) and the three of diamonds ($3\diamondsuit$).

