



Problem of the Week

Grade 7 and 8

What's Your Angle I?

Solution

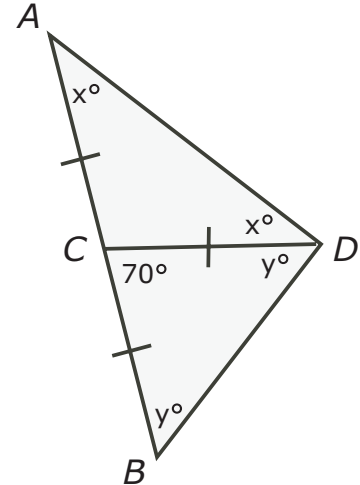
Problem

In $\triangle ABD$, C is on AB such that $AC = CB = CD$ and $\angle BCD = 70^\circ$. Determine the measure of $\angle ADB$.

Solution

Since ACB is a straight line, $\angle ACD + \angle DCB = 180^\circ$ but $\angle DCB = 70^\circ$ so $\angle ACD = 110^\circ$.

In $\triangle ACD$, since $AC = CD$, $\triangle ACD$ is isosceles and $\angle CAD = \angle CDA = x^\circ$.



The angles in a triangle sum to 180° so in $\triangle ACD$

$$\begin{aligned}\angle CAD + \angle CDA + \angle ACD &= 180^\circ \\ x^\circ + x^\circ + 110^\circ &= 180^\circ \\ 2x &= 70 \\ x &= 35\end{aligned}$$

Similarly, in $\triangle BCD$, since $CB = CD$, $\triangle BCD$ is isosceles and $\angle CBD = \angle CDB = y^\circ$.

The angles in a triangle sum to 180° so in $\triangle CBD$

$$\begin{aligned}\angle CBD + \angle CDB + \angle BCD &= 180^\circ \\ y^\circ + y^\circ + 70^\circ &= 180^\circ \\ 2y &= 110 \\ y &= 55\end{aligned}$$

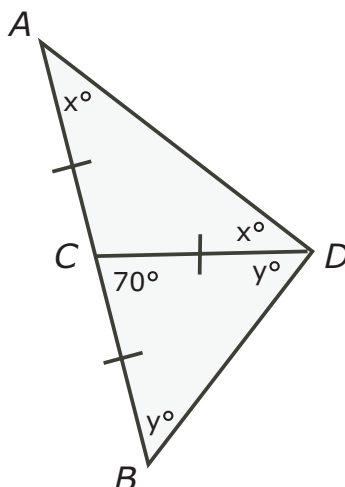
Then $\angle ADB = \angle CDA + \angle CDB = x^\circ + y^\circ = 35^\circ + 55^\circ = 90^\circ$.

$\therefore$ the measure of $\angle ADB$ is 90° .

See the next page for an interesting idea.



It turns out that it is not necessary to find the angles in the problem.



Here is a second solution to the problem.

In $\triangle CAD$, since $CA = CD$, $\triangle CAD$ is isosceles and $\angle CAD = \angle CDA = x^\circ$.

In $\triangle CBD$, since $CB = CD$, $\triangle CBD$ is isosceles and $\angle CBD = \angle CDB = y^\circ$.

The angles in a triangle sum to 180° so in $\triangle ABD$

$$\begin{aligned}
 \angle BAD + \angle ADB + \angle ABD &= 180^\circ \\
 x^\circ + (x^\circ + y^\circ) + y^\circ &= 180^\circ \\
 2x^\circ + 2y^\circ &= 180^\circ \\
 x^\circ + y^\circ &= 90^\circ
 \end{aligned}$$

But $\angle ADB = \angle ADC + \angle CDB = x^\circ + y^\circ = 90^\circ$.

$\therefore$ the measure of $\angle ADB$ is 90° .

