

## Problem of the Week

### Grade 11 and 12

### What's Your Angle Anyway III? Solution

#### Problem

$AB$  is a diameter of a circle with centre  $C$ .  $D$  is a point on the circumference of the circle other than  $A$  and  $B$ . Determine the measure of  $\angle ADB$ .

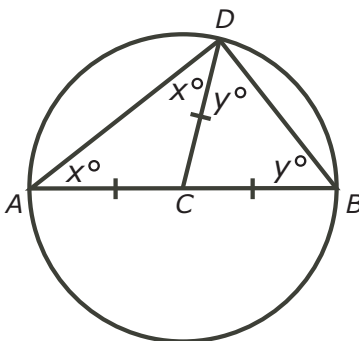
#### Solution

Join  $D$  to the centre  $C$ . Since  $CA$ ,  $CB$  and  $CD$  are radii of the circle,  $CA = CB = CD$ .

Since  $CA = CD$ ,  $\triangle CAD$  is isosceles and  $\angle CAD = \angle CDA = x^\circ$ .

Since  $CB = CD$ ,  $\triangle CBD$  is isosceles and  $\angle CBD = \angle CDB = y^\circ$ .

This new information is marked on the following diagram.



The angles in a triangle add to  $180^\circ$  so in  $\triangle ABD$

$$\begin{aligned}
 \angle ADB + \angle DAB + \angle DBA &= 180^\circ \\
 (x^\circ + y^\circ) + x^\circ + y^\circ &= 180^\circ \\
 2(x^\circ + y^\circ) &= 180^\circ \\
 x^\circ + y^\circ &= 90^\circ
 \end{aligned}$$

But  $\angle ADB = x^\circ + y^\circ$  so  $\angle ADB = 90^\circ$ .

This result is often expressed as a theorem for circles:

*An angle ( $\angle ADB$ ) inscribed in a circle by the diameter ( $AB$ ) of a circle is  $90^\circ$ .*

