**Canadian Mathematics Competition**

*An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario*

Grade 8 solutions follow the Grade 7 solutions

# 2008 Gauss Contests

(Grades 7 and 8)

**Wednesday, May 14, 2008**

*Solutions*

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**Grade 7**

Grade 8 solutions follow the Grade 7 solutions

1. Calculating, $6 \times 2 - 3 = 12 - 3 = 9$.

   Answer: (A)

2. Calculating, $1 + 0.01 + 0.0001 = 1.01 + 0.0001 = 1.0100 + 0.0001 = 1.0101$.

   Answer: (E)

3. Using a common denominator of 8, we have $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8}$.

   Answer: (D)

4. Since the polygon has perimeter 108 cm and each side has length 12 cm, then the polygon has $108 \div 12 = 9$ sides.

   Answer: (D)

5. In the set, three of the numbers are greater than or equal to 3, and two of the numbers are less than 3.
   The smallest number must be one of the numbers that is less than 3, that is, 2.3 or 2.23.
   Of these two numbers, 2.23 is the smallest, so is the smallest number in the set.

   Answer: (D)

6. Since $PQ$ is a straight line, then $x^\circ + x^\circ + x^\circ + x^\circ + x^\circ = 180^\circ$ or $5x = 180$ or $x = 36$.

   Answer: (A)

7. 20 is not a prime number, since it is divisible by 2.
   21 is not a prime number, since it is divisible by 3.
   25 is not a prime number, since it is divisible by 5.
   27 is not a prime number, since it is divisible by 3.
   23 is a prime number, since its only positive divisors are 1 and 23.

   Answer: (C)

8. Kayla walked 8 km on Monday.
   On Tuesday, she walked $8 \div 2 = 4$ km.
   On Wednesday, she walked $4 \div 2 = 2$ km.
   On Thursday, she walked $2 \div 2 = 1$ km.
   On Friday, she walked $1 \div 2 = 0.5$ km.

   Answer: (E)

9. Since 50% selected chocolate and 10% selected strawberry as their favourite flavour, then overall $50\% + 10\% = 60\%$ chose chocolate or strawberry as their favourite flavour.
   Now $60\% = \frac{60}{100} = \frac{3}{5}$, so $\frac{3}{5}$ of the people surveyed selected chocolate or strawberry as their favourite flavour.

   Answer: (A)

10. Since Max sold 41 glasses of lemonade on Saturday and 53 on Sunday, he sold $41 + 53 = 94$ glasses in total.
    Since he charged 25 cents for each glass, then his total sales were $94 \times \$0.25 = \$23.50$.

    Answer: (A)

11. Since Chris spent \$68 in total and \$25 on the helmet, then he spent $\$68 - \$25 = \$43$ on the two hockey sticks.
Since the two sticks each cost the same amount, then this cost was $\$43 \div 2 = \$21.50$.

Answer: (C)

12. The number below and between 17 and 6 is $17 - 6 = 11$.
The number below and between 8 and 11 is $11 - 8 = 3$.
The number below and between 11 and 2 is $11 - 2 = 9$.
The number below and between 7 and 3 is $7 - 3 = 4$.
The number below and between 3 and 9 is $9 - 3 = 6$.

$$\begin{array}{ccccccccc} 8 & & 9 & & 17 & & 6 & & 4 \\ & 1 & & 8 & & 11 & & 2 & \\ & & 7 & & 3 & & 9 & & \\ & & & 4 & & 6 & & & \\ & & & & x & & & & \end{array}$$

Therefore, $x = 6 - 4 = 2$.

Answer: (B)

13. Since $PQ = PR$, then $\angle PQR = \angle PRQ$.
Since the angles in a triangle add up to $180^\circ$, then $40^\circ + \angle PQR + \angle PRQ = 180^\circ$, so $\angle PQR + \angle PRQ = 140^\circ$.
Since $\angle PQR = \angle PRQ$, then $\angle PQR = \angle PRQ = 70^\circ$.
Since the angle labelled as $x^\circ$ is opposite $\angle PRQ$, then $x^\circ = \angle PRQ = 70^\circ$, so $x = 70$.

Answer: (B)

14. The sum of Wesley's and Breenah's ages is 22.
After each year, each of their ages increases by 1, so the sum of their ages increases by 2.
For the sum to increase from 22 to $2 \times 22 = 44$, the sum must increase by 22, which will take $22 \div 2 = 11$ years.

Answer: (E)

15. The first transformation is a $180^\circ$ rotation of the letter, which gives G $\rightarrow$ ⅁.
The second transformation is a reflection across a vertical axis, which gives G $\rightarrow$ ⅁ $\rightarrow$ ⅁.

Answer: (D)

16. In the diagram, the length of one side of the large square is equal to eight side lengths of the smaller squares, so the large square consists of $8 \times 8 = 64$ small squares.
Of these 64 small squares, 48 are shaded. (We can obtain this number by counting the 48 shaded squares or by counting the 16 *unshaded* squares.)
As a percentage, this fraction equals $\frac{48}{64} \times 100\% = \frac{3}{4} \times 100\% = 75\%$.

Answer: (D)

17. *Solution 1*
Since the perimeter of a rectangle equals twice the length plus twice the width, then the length plus the width equals $120 \div 2 = 60$.
Since the length equals twice the width plus 6, then twice the width plus the width equals $60 - 6 = 54$.
In other words, three times the width equals 54, so the width equals $54 \div 3 = 18$.

*Solution 2*
Let the width of the rectangle be $w$.
Then the length of the rectangle is $2w+6$.
Since the perimeter is 120, then

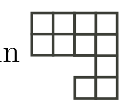
$$\begin{aligned} 2w+2(2w+6) &= 120 \\ 2w+4w+12 &= 120 \\ 6w &= 108 \\ w &= 18 \end{aligned}$$

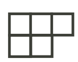
so the width is 18.

Answer: (B)

18. The sum of Rishi's marks so far is $71+77+80+87=315$.
Since Rishi's mark on his next test is between 0 and 100, the sum of his marks will be between $315+0=315$ and $315+100=415$ after his next test.
Since his average equals the sum of his marks divided by the number of marks, then his average will be between $\frac{315}{5}=63$ and $\frac{415}{5}=83$.
Of the given choices, the only one in this range is 82.

Answer: (C)

19. After some experimentation, the only way in which the two given pieces can be put together to stay within a $4\times 4$ grid and so that one of the given choices can fit together with them is to rotate the second piece by $90^\circ$ clockwise, and combine to obtain  .

Therefore, the missing piece is  .

Answer: (C)

20. The possible ways of writing 72 as the product of three different positive integers are: $1\times2\times36$; $1\times3\times24$; $1\times4\times18$; $1\times6\times12$; $1\times8\times9$; $2\times3\times12$; $2\times4\times9$; $3\times4\times6$.
(We can find all of these possibilities systematically by starting with the smallest possible first number and working through the possible second numbers, then go to the next possible smallest first number and continue.)
The sums of these sets of three numbers are 39, 28, 23, 19, 18, 17, 15, 13, so the smallest possible sum is 13.

Answer: (A)

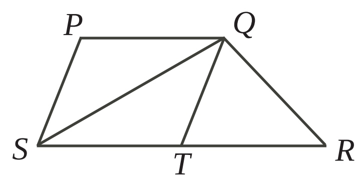
21. Since Andrea has completed $\frac{3}{7}$ of the total 168 km, then she has completed $\frac{3}{7}\times 168$ km or $3\times 24=72$ km.
This means that she has $168-72=96$ km remaining.
To complete the 96 km in her 3 remaining days, she must average $\frac{96}{3}=32$ km per day.

Answer: (D)

22. *Solution 1*
Since $PQ$ is parallel to $SR$, then the height of $\triangle PQS$ (considering $PQ$ as the base) and the height of $\triangle SRQ$ (considering $SR$ as the base) are the same (that is, the vertical distance between $PQ$ and $SR$).

Since $SR$ is twice the length of $PQ$ and the heights are the same, then the area of $\triangle SRQ$ is twice the area of $\triangle PQS$.
In other words, the area of $\triangle PQS$ is $\frac{1}{3}$ of the total area of the trapezoid, or $\frac{1}{3} \times 12 = 4$.

*Solution 2*
Draw a line from $Q$ to $T$, the midpoint of $SR$.




Since $SR = 2(PQ)$ and $T$ is the midpoint of $SR$, then $PQ = ST = TR$.
We consider $PQ$, $ST$ and $TR$ as the bases of $\triangle PQS$, $\triangle STQ$ and $\triangle TRQ$, respectively.
Using these three segments as the bases, each of these triangles has the same height, since $PQ$ is parallel to $SR$.
Since $PQ = ST = TR$ and these triangles have the same height, then the three triangles each have the same area.
The trapezoid is thus cut into three triangles of equal area.
Therefore, the area of $\triangle PQS$ is one-third of the area of entire trapezoid, or $\frac{1}{3} \times 12 = 4$.

ANSWER: (B)

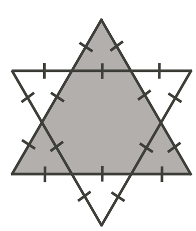
23. Since Ethan does not sit next to Dianne, the four must arrange themselves in one of the configurations:

| | | |
|---|---|---|
| D __ E __ | D __ __ E | __ D __ E |
| E __ D __ | E __ __ D | __ E __ D |

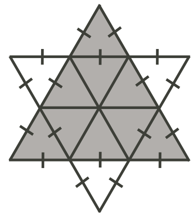
For each of these six configurations, there are two ways for Beverly and Jamaal to sit (either with Beverly on the left or with Jamaal on the left).
Therefore, there are $6 \times 2 = 12$ possible ways that the four can sit. (Try listing them out!)

ANSWER: (B)

24. Since the two large triangles are equilateral, then each of their three angles equals $60^\circ$.
Therefore, each of 6 small triangles in the star has an angle of $60^\circ$ between the two equal sides.
But each of these 6 small triangles is isosceles so each of the remaining two angles must equal $\frac{1}{2}(180^\circ - 60^\circ)$ or $60^\circ$.
Therefore, each of the small triangles is equilateral.



This shows us that the inner hexagon has all sides equal, and also that each angle is $180^\circ - 60^\circ$ or $120^\circ$, so the hexagon is regular.
Next, we draw the three diagonals of the hexagon that pass through its centre (this is possible because of the symmetry of the hexagon).



Also, because of symmetry, each of the angles of the hexagon is split in half, to get $120^\circ \div 2 = 60^\circ$. Therefore, each of the 6 new small triangles has two $60^\circ$ angles, and so must have its third angle equal to $60^\circ$ as well. Thus, each of the 6 new small triangles is equilateral.
So all 12 small triangles are equilateral. Since each has one side length marked by a single slash, then these 12 small triangles are all identical.
Since the total area of the star is 36, then the area of each small triangle is $36 \div 12 = 3$.
Since the shaded area is made up of 9 of these small triangles, its area is $9 \times 3 = 27$.

Answer: (C)

25. First we look at the integers from 2000 to 2008.
Since we can ignore the 0s when adding up the digits, the sum of all of the digits of these integers is

$$2 + (2 + 1) + (2 + 2) + (2 + 3) + (2 + 4) + (2 + 5) + (2 + 6) + (2 + 7) + (2 + 8) = 54$$

Next, we look at the integers from 1 to 1999.
Again, since we can ignore digits of 0, we consider these numbers as 0001 to 1999, and in fact as the integers from 0000 to 1999, including 0000 to make 2000 integers in total.
Of these 2000 integers, 200 have a units digit of 0, 200 have a units digit of 1, and so on.
(One integer out of every 10 has a units digit of 0, and so on.)
Therefore, the sum of the units digits of these integers is

$$200(0)+200(1)+\cdots+200(8)+200(9) = 200+400+600+800+1000+1200+1400+1600+1800 = 9000$$

Of these 2000 integers, 200 have a tens digit of 0, 200 have a tens digit of 1, and so on.
(Ten integers out of every 100 have a tens digit of 0, and so on.)
Therefore, the sum of the tens digits of these integers is

$$200(0) + 200(1) + \cdots + 200(8) + 200(9) = 9000$$

Of these 2000 integers, 200 have a hundreds digit of 0 (that is, 0000 to 0099 and 1000 to 1099), 200 have a hundreds digit of 1, and so on.
(One hundred integers out of every 1000 have a hundreds digit of 0, and so on.)
Therefore, the sum of the hundreds digits of these integers is

$$200(0) + 200(1) + \cdots + 200(8) + 200(9) = 9000$$

Of these 2000 integers, 1000 have a thousands digit of 0 and 1000 have a thousands digits of 1.
Therefore, the sum of the thousands digits of these integers is

$$1000(0) + 1000(1) = 1000$$

Overall, the sum of all of the digits of these integers is $54 + 9000 + 9000 + 9000 + 1000 = 28\,054$.

Answer: (E)

**Grade 8**

1. Using the correct order of operations, $8 \times (6 - 4) + 2 = 8 \times 2 + 2 = 16 + 2 = 18$.

   Answer: (C)

2. Since the polygon has perimeter 108 cm and each side has length 12 cm, then the polygon has $108 \div 12 = 9$ sides.

   Answer: (D)

3. Since $\angle PQR = 90^\circ$, then $2x^\circ + x^\circ = 90^\circ$ or $3x = 90$ or $x = 30$.

   Answer: (A)

4. Calculating, $(1 + 2)^2 - (1^2 + 2^2) = 3^2 - (1 + 4) = 9 - 5 = 4$.

   Answer: (B)

5. When these four numbers are listed in increasing order, the two negative numbers come first, followed by the two positive numbers.
   Of the two positive numbers, 0.28 and 2.8, the number 0.28 is the smallest.
   Of the two negative numbers, $-0.2$ and $-8.2$, the number $-8.2$ is the smallest.
   Therefore, the correct order is $-8.2$, $-0.2$, 0.28, 2.8.

   Answer: (A)

6. From the given formula, the number that should be placed in the box is $5^3+5-1 = 125+4 = 129$.

   Answer: (E)

7. Since 50% selected chocolate and 10% selected strawberry as their favourite flavour, then overall $50\% + 10\% = 60\%$ chose chocolate or strawberry as their favourite flavour.
   Now $60\% = \frac{60}{100} = \frac{3}{5}$, so $\frac{3}{5}$ of the people surveyed selected chocolate or strawberry as their favourite flavour.

   Answer: (A)

8. *Solution 1*
   Since 5 times the number minus 9 equals 51, then 5 times the number must equal 60 (that is, $51 + 9$).
   Therefore, the original number is 60 divided by 5, or 12.

   *Solution 2*
   Let the original number be $x$.
   Then $5x - 9 = 51$, so $5x = 51 + 9 = 60$, so $x = \dfrac{60}{5} = 12$.

   Answer: (D)

9. *Solution 1*
   Since Danny weighs 40 kg, then 20% of his weight is $\frac{20}{100} \times 40 = \frac{1}{5} \times 40 = 8$ kg.
   Since Steven weighs 20% more than Danny, his weight is $40 + 8 = 48$ kg.

   *Solution 2*
   Since Steven weighs 20% more than Danny, then Steven's weight is 120% of Danny's weight.
   Since Danny's weight is 40 kg, then Steven's weight is $\frac{120}{100} \times 40 = \frac{6}{5} \times 40 = 48$ kg.

   Answer: (C)

10. Of the given 11 numbers, the numbers 3, 5, 7, 11 and 13 are prime. (4, 6, 8, 10 and 12 are not prime, since they are divisible by 2, and 9 is not prime since it is divisible by 3.)
Therefore, 5 of the 11 numbers are prime.
Thus, if a card is chosen at random and flipped over, the probability that the number on this card is a prime number is $\frac{5}{11}$.

Answer: (E)

11. In centimetres, the dimensions of the box are 20 cm, 50 cm, and 100 cm (since 1 m equals 100 cm).
Therefore, the volume of the box is

$$(20 \text{ cm}) \times (50 \text{ cm}) \times (100 \text{ cm}) = 100\,000 \text{ cm}^3$$

Answer: (D)

12. *Solution 1*
Since each pizza consists of 8 slices and each slice is sold for \$1, then each pizza is sold for \$8 in total.
Since 55 pizzas are sold, the total revenue is $55 \times \$8 = \$440$.
Since 55 pizzas were bought initially, the total cost was $55 \times \$6.85 = \$376.75$.
Therefore, the total profit was $\$440 - \$376.75 = \$63.25$.

*Solution 2*
Since each pizza consists of 8 slices and each slice is sold for \$1, then each pizza is sold for \$8 in total.
Since each pizza was bought for \$6.85 initially, then the school makes a profit of $\$8.00 - \$6.85$ or \$1.15 per pizza.
Since the school completely sold 55 pizzas, then its total profit was $55 \times \$1.15 = \$63.25$.

Answer: (D)

13. Since $RSP$ is a straight line, then $\angle RSQ + \angle QSP = 180^\circ$, so $\angle RSQ = 180^\circ - 80^\circ = 100^\circ$.
Since $\triangle RSQ$ is isosceles with $RS = SQ$, then

$$\angle RQS = \tfrac{1}{2}(180^\circ - \angle RSQ) = \tfrac{1}{2}(180^\circ - 100^\circ) = 40^\circ$$

Similarly, since $\triangle PSQ$ is isosceles with $PS = SQ$, then

$$\angle PQS = \tfrac{1}{2}(180^\circ - \angle PSQ) = \tfrac{1}{2}(180^\circ - 80^\circ) = 50^\circ$$

Therefore, $\angle PQR = \angle PQS + \angle RQS = 50^\circ + 40^\circ = 90^\circ$.

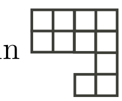
Answer: (B)

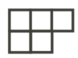
14. On Monday, Amos read 40 pages.
On Tuesday, Amos read 60 pages, for a total of $40 + 60 = 100$ pages so far.
On Wednesday, Amos read 80 pages, for a total of $100 + 80 = 180$ pages so far.
On Thursday, Amos read 100 pages, for a total of $180 + 100 = 280$ pages so far.
On Friday, Amos read 120 pages, for a total of $280 + 120 = 400$ pages so far.
Therefore, Amos finishes the 400 page book on Friday.

Answer: (A)

15. If Abby had 23 nickels, the total value would be $23 \times \$0.05 = \$1.15$.
But the total value of Abby's coins is \$4.55, which is \$3.40 more.
Since a quarter is worth 20 cents more than a nickel, then every time a nickel is replaced by a quarter, the total value of the coins increase by 20 cents.
For the total value to increase by \$3.40, we must replace $\$3.40 \div \$0.20 = 17$ nickels with quarters.
Therefore, Abby has 17 quarters.
(To check, if Abby has 17 quarters and 6 nickels, the total value of the coins that she has is $17 \times \$0.25 + 6 \times \$0.05 = \$4.25 + \$0.30 = \$4.55$.)

ANSWER: (B)

16. After some experimentation, the only way in which the two given pieces can be put together to stay within a $4 \times 4$ grid and so that one of the given choices can fit together with them is to rotate the second piece by $90^\circ$ clockwise, and combine to obtain  .

Therefore, the missing piece is  .

ANSWER: (C)

17. The digits after the decimal point occur in repeating blocks of 6 digits.
Since $2008 \div 6 = 334.666...$, then the 2008th digit after the decimal point occurs after 334 blocks of digits have been used.
In 334 blocks of 6 digits, there are $334 \times 6 = 2004$ digits in total. Therefore, the 2008th digit is 4 digits into the 335th block, so must be 8.

ANSWER: (A)

18. Since Andrea has completed $\frac{3}{7}$ of the total 168 km, then she has completed $\frac{3}{7} \times 168$ km or $3 \times 24 = 72$ km.
This means that she has $168 - 72 = 96$ km remaining.
To complete the 96 km in her 3 remaining days, she must average $\frac{96}{3} = 32$ km per day.

ANSWER: (D)

19. *Solution 1*
After some trial and error, you might discover that $x = 0$ and $y = 7$ works, since
$307 + 703 = 1010$.
Therefore, since we are asked for the unique value of $y - x$, it must be $7 - 0 = 7$.

*Solution 2*
When performing this addition, in the units column either $y + 3 = x$ or $y + 3 = x$ with a carry of 1, meaning that $y + 3 = 10 + x$.
Therefore, either $y - x = -3$ or $y - x = 10 - 3 = 7$.
If there was no carry, then adding up the tens digits, we would get $x + x$ ending in a 1, which is impossible as $x + x = 2x$ which is even.
Therefore, the addition $y + 3$ must have a carry of 1.
Therefore, $y - x = 7$.

ANSWER: (C)

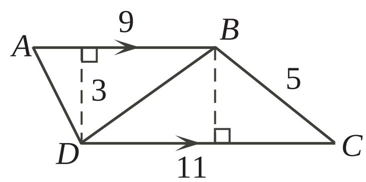
20. *Solution 1*

The area of a trapezoid equals one-half times the sum of the bases times the height.
Therefore, the area of this trapezoid is

$$\tfrac{1}{2} \times (9 + 11) \times 3 = \tfrac{1}{2} \times 20 \times 3 = 10 \times 3 = 30$$

*Solution 2*

We draw diagonal $BD$.

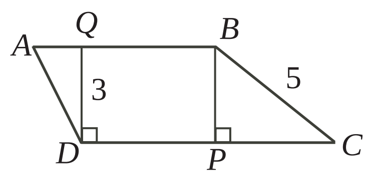



$\triangle ABD$ has a base of 9 and a height of 3.
$\triangle BCD$ has a base of 11 and a height of 3.
The area of the trapezoid is equal to the sum of the areas of these two triangles, or

$$\tfrac{1}{2} \times 9 \times 3 + \tfrac{1}{2} \times 11 \times 3 = \tfrac{27}{2} + \tfrac{33}{2} = \tfrac{60}{2} = 30$$

*Solution 3*

Draw perpendicular lines from $B$ to $CD$ and from $D$ to $AB$, as shown.




By the Pythagorean Theorem, $BP^2 + PC^2 = BC^2$ or $3^2 + PC^2 = 5^2$ or $PC^2 = 25 - 9 = 16$, so $PC = 4$.
Since $DC = 11$, then $DP = 11 - 4 = 7$.
Since $QBPD$ is a rectangle, then $QB = DP = 7$, so $AQ = 9 - 7 = 2$.
The area of trapezoid $ABCD$ equals the sum of the area of $\triangle AQD$, rectangle $QBPD$ and $\triangle BPC$, or

$$\tfrac{1}{2} \times 2 \times 3 + 7 \times 3 + \tfrac{1}{2} \times 4 \times 3 = 3 + 21 + 6 = 30$$

Answer: (D)

21. The object has 7 "front" faces, each of which is $1 \times 1$. Therefore, the surface area of the front is $7 \times 1 \times 1 = 7$.
Similarly, the surface area of the "back" is 7.
Now consider the faces on the left, top, right and bottom. Each of these faces is $1 \times 2$, so each has an area of 2.
How many of these faces are there?
If we start at the bottom left and travel clockwise around the figure, we have 2 left faces, 2 top faces, 2 left faces, 1 top face, 3 right faces, 1 bottom face, 1 right face, and 2 bottom faces, or 14 faces in total.
Therefore, the surface area accounted for by these faces is $14 \times 2 = 28$.
Therefore, the total surface area of the object is $7 + 7 + 28 = 42$.

Answer: (A)

22. There are 6 possibilities for the first row of the grid:

$$1,2,3 \quad 1,3,2 \quad 2,1,3 \quad 2,3,1 \quad 3,1,2 \quad 3,2,1$$

Consider the first row of 1, 2, 3:

| | | |
|---|---|---|
| 1 | 2 | 3 |
| | | |
| | | |

.

The first column could be 1, 2, 3 or 1, 3, 2:

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 2 | | |
| 3 | | |

or

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 3 | | |
| 2 | | |

.

Each of these grids can be finished with the given rules, but can only be finished in one way. (In the first grid, the middle number in the bottom row cannot be 2 or 3, so is 1, so the middle number in the middle row is 3, so the right column is 3, 1, 2.
Similarly, in the second grid, the middle number in the middle row must be 1. Try completing this grid!)
Therefore, a first row of 1, 2, 3 gives two possible grids.
Similarly, each of the other 5 possible first rows will give two other grids.
(We can see this by trying each of these possibilities or by for example switching all of the 2s and 3s to get the grids with a first row of 1, 3, 2.)
Therefore, the total number of different ways of filling the grid is $6 \times 2 = 12$.

Answer: (B)

23. Since the area of the larger circle is $64\pi$ and each circle is divided into two equal areas, then the larger shaded area is $\frac{1}{2}$ of $64\pi$, or $32\pi$.
Let $r$ be the radius of the larger circle.
Since the area of the larger circle is $64\pi$, then $\pi r^2 = 64\pi$ or $r^2 = 64$ or $r = \sqrt{64} = 8$, since $r > 0$.
Since the smaller circle passes through the centre of the larger circle and just touches the outer circle, then by symmetry, its diameter must equal the radius of the larger circle. (In other words, if we join the centre of the larger circle to the point where the two circles just touch, this line will be a radius of the larger circle and a diameter of the smaller circle.)
Therefore, the diameter of the smaller circle is 8, so its radius is 4.
Therefore, the area of the smaller circle is $\pi(4^2) = 16\pi$, so the smaller shaded area is $\frac{1}{2} \times 16\pi$ or $8\pi$.
Therefore, the total of the shaded areas is $32\pi + 8\pi = 40\pi$.

Answer: (D)

24. First we look at the integers from 2000 to 2008.
Since we can ignore the 0s when adding up the digits, the sum of all of the digits of these integers is

$$2 + (2+1) + (2+2) + (2+3) + (2+4) + (2+5) + (2+6) + (2+7) + (2+8) = 54$$

Next, we look at the integers from 1 to 1999.
Again, since we can ignore digits of 0, we consider these numbers as 0001 to 1999, and in fact as the integers from 0000 to 1999.
Of these 2000 integers, 200 have a units digit of 0, 200 have a units digit of 1, and so on.
(One integer out of every 10 has a units digit of 0, and so on.)
Therefore, the sum of the units digits of these integers is

$$200(0) + 200(1) + \cdots + 200(8) + 200(9) = 200(0+1+2+3+4+5+6+7+8+9) = 200(45) = 9000$$

Of these 2000 integers, 200 have a tens digit of 0, 200 have a tens digit of 1, and so on.
(Ten integers out of every 100 have a tens digit of 0, and so on.)
Therefore, the sum of the tens digits of these integers is

$$200(0) + 200(1) + \cdots + 200(8) + 200(9) = 9000$$

Of these 2000 integers, 200 have a hundreds digit of 0 (that is, 0000 to 0099 and 1000 to 1099), 200 have a hundreds digit of 1, and so on.
(One hundred integers out of every 1000 have a hundreds digit of 0, and so on.)
Therefore, the sum of the hundreds digits of these integers is

$$200(0) + 200(1) + \cdots + 200(8) + 200(9) = 9000$$

Of these 2000 integers, 1000 have a thousands digit of 0 and 1000 have a thousands digits of 1.
Therefore, the sum of the thousands digits of these integers is

$$1000(0) + 1000(1) = 1000$$

Overall, the sum of all of the digits of these integers is $54 + 9000 + 9000 + 9000 + 1000 = 28\,054$.

ANSWER: (E)

25. Since the length of the candles was equal at 9 p.m., the longer one burned out at 10 p.m., and the shorter one burned out at midnight, then it took 1 hour for the longer candle and 3 hours for the shorter candle to burn this equal length.
Therefore, the longer candle burned 3 times as quickly as the shorter candle.
Suppose that the shorter candle burned $x$ cm per hour.
Then the longer candle burned $3x$ cm per hour.
From its lighting at 3 p.m. to 9 p.m., the longer candles burned for 6 hours, so burned $6 \times 3x$ or $18x$ cm.
From its lighting at 7 p.m. to 9 p.m., the shorter candle burned for 2 hours, so burns $2 \times x = 2x$ cm.
But, up to 9 p.m., the longer candle burned 32 cm more than the shorter candle, since it began 32 cm longer.
Therefore, $18x - 2x = 32$ or $16x = 32$ or $x = 2$.
In summary, the shorter candle burned for 5 hours at 2 cm per hour, so its initial length was 10 cm.
Also, the longer candle burned for 7 hours at 6 cm per hour, so its initial length was 42 cm.
Thus, the sum of the original lengths is $42 + 10 = 52$ cm.

ANSWER: (E)